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Thermodynamics

A NUMERICAL METHOD TO STUDY THE HEAT TRANSFER IN ROCKET NOZZLE THROAT BY INVERSE HEAT CONDUCTION TECHNIQUE

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ABSTRACT

This paper is study of one dimensional heat conduction with thermo- physical properties K, ROW, Cp of a material varying with temperature. The physical problem is characterized by a slab of infinite length and thickness L, imposed with a net heat flux at x=0, with the other end being insulated. The problem is solved by using Inverse Heat Conduction Technique (IHCP).

Keywords: Duhamel's theorem, Heat Conduction, Sensitivity Coefficient, Taylor series, tridiogonal system.

LITERATURE SURVEY

Function Estimation verses Parameter estimation

The word function estimation is used in connection with heat flux. Heat flux is found to be an arbitrary, single valued function of time. Heat flux can be positive, negative, constant or abruptly changing periodic or not and so on. It may be influenced by human decisions. For example, a pilot of a shuttle can change the reentry trajectory. In IHCP problem, the surface heat flux is a function of time and may require hundreds of individually estimated heat flux components, q, to define it adequately. Related estimation problems are called parameter estimation problems which are inverse problems but with emphasis on the estimation of certain parameters or constants or physical properties. In the context of heat conduction, one might be interested in determining the thermal conductivity of a body based upon internal temperature histories and the surface heat flux and other boundary conditions. The thermal conductivity of iron near room temperature, for example, could be a parameter. It is not a function and does not require hundreds of values of K to describe it. The parameter estimation and function estimation problems start to merge if estimates are made of the thermal conductivity, K as a function of temperature. However, K(T) function is not arbitrary and is not adjustable by humans.

Measurements

In the IHCP problem, there are a number of measured quantities in addition to temperature such as time, sensor location and specimen thickness. Each is assumed to be accurately known except the temperature. If this is not true, then it may be necessary, for example to simultaneously estimate sensor location and the surface heat flux. The latter problem would involve both inverse heat conduction and parameter estimation problem.

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If the thermal properties are not accurately known, they should be determined as accurately as possible using parameter estimation technique. The temperature measurements are assumed to contain the major sources of error or uncertainty. Any known systematic effective due to calibration errors, presence of sensors, conduction or convection losses or whatever is assumed to be removed to the extent that the remaining errors may be considered at random. These random errors can statistically be described.

Why is IHCP Difficult?

The IHCP is difficult because it is extremely sensitive to measurement errors. The difficulties are pronounced as one tries to obtain maximum amount of information from the data. For the one dimensional IHCP, when discrete values of q curve are estimated, maximizing the amount of information implies small time steps between q values.

However , uses of small time steps frequently introduces instability in the e solution of the IHCP unless restrictions are employed. The conditions of small time steps have the opposite effects in the IHCP compared to the numerical solution of the heat conduction equation. In the latter, the stability problems often can be corrected by reducing the size of time steps.

Damping and lagging effects

The transient temperature response of an internal point in an opaque heat conducting body is quite different from that of a point at the surface. The internal temperature excursions are much diminished internally compared to the surface changes. This is a damping effect. A large time delay or lag in the internal response can also be noted. These lagging and damping effects for the direct problem are important because they provide engineering insight into the difficulties encountered in the inverse problem.

Classification of methods

The methods for solving the inverse conduction problem can be classified in several ways, some of which are discussed in this section. One classification relates to the ability of a method to treat linear as well as non-linear IHCPs. The two basic procedures given herein are the function specification and regularization methods. If the heat flux is varying with time, the method of solving IHCP is by function specification. The regularization method is a procedure which modifies the least square approach by adding factors that are intended to reduce excursions in the unknown function, such as the surface heat flux.

The method of solution of the heat conduction equation is another way to classify the IHCP. Methods of solution include the use of Duhamel's theorem, Finite elements and Finite control volumes. The use of Duhamel's theorem restricts the IHCP algorithm to a linear case; where as other two procedures can treat the non-linear case also.

The domain used in IHCP can also be used to classify the method of solution. Three time domains have been proposed (1) only the present time (2) to the present time plus few time steps and (#) the complete time domain. The last classification to be mentioned is related to the dimensionality of IHCP. If a single heat flux history is to be determined, the IHCP is considered as one dimensional. In the use of Duhamel's theorem, the physical dimensions are not of concern i.e. the same procedure is used for physically one, two or three dimension bodies provided a single heat flux history is to be estimated. If two or more heat flux histories are estimated and Duhamel's theorem is used, the problem is multidimensional. When the Finite difference or other methods are used for non-linear, the dimensionality of the problem depends on the number of space coordinates needed to describe a heat conducting body, one coordinate would give one dimension problem and so on.

Sensitivity Coefficients

In function estimation as in parameter estimation, a detailed examination of sensitivity coefficients can provide considerable insight into the estimation problem. These coefficients can show the possible areas of difficulty and also lead to experimental design. The sensitivity coefficients are defined as first derivative of the dependent variable such as a heat flux component. If the sensitivity coefficients are either small or correlated to one another, the estimation problem is difficult and very sensitive to estimation errors.

For the IHCP problem, the sensitivity coefficients of interest are those of the first derivatives of temperature T, at location x and time t, with respect to a heat flux component q, and are defined by

$$X j_m (x_j, t_i) = \partial(x_j, t_j)/ \partial q_m$$

For j = 1, 2 3 …………………n, and m = 1, 2 3,……..n

Note that the number of times t, equals the number of heat flux components. If there is only one interior location, that is, j=1, the sensitivity coefficient is simply given by:

$$X_m (t_i) = \partial(T_I)/ \partial q_m$$

For the transient problems considered in the IHCP, the sensitivity coefficients are zero for m>i. In other words, the temperature at t is independent of yet to occur future heat flux component of q_m, m>i. On the way to determine the linearity of an estimation problem, one need to inspect the sensitivity coefficients. If the sensitivity coefficients are not functions of the parameters, then the estimation problem is linear. If they are, then the problem is non- linear. For example, consider the equation

$\partial T(\theta, t)/ \partial q_c = (L/K).2.(\alpha T/ \Pi L^2)^{1/2}$, T<+0.3 which is independent of q_c. Thus, it can be considered to be linear.

Sensitivity Coefficient approach for exactly matching data from a single sensor

A single temperature sensor is considered to be located at a depth x, below the active surface. If the heat flux q_m is constant over the time interval $t_{m-1}<t< t_m$, the value of q_m that forces a matching of computed temperature at x with the measured temperature can be calculated.

The temperature field T(x,t) depends in a continues manner on the unknown heat flux q_m. This dependence is written as $T(x,t,t_{m-1},q_{m-1},q_m)$ where q_{m-1} is the vector of all previous heat values and t indicates the time that the heat flux step begins. Because the temperature field is continues in q_m, it can be extended in Taylor series about an arbitrary but known value of q*.

For linear problems, only the first derivative is non zero, thus the following is an exact result for location x at time t

$T(x,t_{m-1},q_{m-1}, q_m) = T(x,t_m,t_{m-1},q_{m-1,}q^*)+ (q_m-q^*)X(x,t_m,t_{m-1},q_{m-1})$ where the sensitivity Coefficient is defined by

$$X(x,t_m,t_{m-1},q_{m-1}) = \partial T(x,t_m,t_{m-1},q_{m-1},q_m)/ \partial q_m.$$

For the IHCP in which sensors are matched exactly, the left hand side of the equation is replaced by the experimental temperature.

Hence $Y_m = T_k + (q_m-q^*)X_{k,1}$

On solving $q_m= q^* +(y_m-T_k)/X_{k,1}$

Where T_k is the temperature at time t, for the sensor node k with q =q* over $t_{m-1}<t< t_m$. The calculation procedure is to assume an arbitrary value of q, calculate the temperature field T and knowing the sensitivity coefficient X, calculate the heat flux that exactly matches the temperature data Y_m. Once q_m is known, the complete field can be calculated.

The algorithm for exactly matching the temperature data from the single sensor can be summarized as follows.

1) Assume an arbitrary value of q* and calculate the entire temperature field.

2) Using the same matrix coefficients an, b_n, c_n, d_n as were used for T calculation of step 1, calculate sensitivity coefficients.

3) Calculate the heat flux that exactly matches the experimental temperature data by using Taylor series expansion

$Q_m = q^* + (Y_m - T_k) / X_{k,1}$

4) Calculate the temperature field from Taylor series expansion

$T_j = T_j + (q_m - q^*) X_{j,1}$.

1.7 Relevant work:

References [1-13] give an insight into non linear Inverse Heat Conduction problems in 1-d & 2D environment.

METHODOLOGY OF WORK

Numerical solution of a non linear Inverse heat conduction problem of a slab with an insulated boundary condition:

Here we consider one dimensional heat conduction with thermo physical properties K, ROW, Cp of a material varying with temperature. The physical problem is that of a slab of infinite length and thickness L, imposed with a net heat flux at x=0, with the other end being insulated.

Consider the slab which is divided into n-1 parts. For every control volume, energy gaining per unit time =energy leaving the control volume- energy entering the control volume.

For the first grid point

$K/\delta x(T_2^{n+1} - T_1^{n+1}) + Q = \rho Cp\ \delta x/2*\delta t(T_1^{n+1} - T_1^{n})$

$T_1^{n+1} (\rho Cp\ \delta x/2*\delta t) + T_2^{n+1} (-K/ \delta x) = T_1^{n+1} (\rho Cp\ \delta x/2*\delta t)+Q$

$A(1) = \rho Cp\ \delta x/2*\delta t$

$B(1)= -K/ \delta x$

$D(1)= \rho Cp\ \delta x/2*\delta t+Q$

For the second to n-1 grid points

$T_i^{n+1} (\rho Cp\ \delta x/\delta t + K/ \delta x) + T_{i+1}^{n+1} (-K/ \delta x) + T_{i-1}^{n+1} (-K/ \delta x) = T_i^{n} (\rho Cp\ \delta x/\delta t)$

$A(i)= \rho Cp\ \delta x/\delta t + K/ \delta x$

$B(i)= -K/ \delta x$

$C(i)= -K/ \delta x$

$D(i) = T_i^{n}(\rho Cp\ \delta x/\delta t)$

For the nth point

$-K/ \delta x(T_n^{n+1} - T_{n-1}^{n+1}) = \rho Cp\ \delta x/2*\delta t(T_n^{n+1} - T_n^{n})$

$T_n^{n+1}(\rho Cp\ \delta x/2*\delta t + K/ \delta x) + T_{n-1}^{n+1}(-K/ \delta x) = \rho Cp\ \delta x/2*\delta t* T_n^{n}$

$A(n)= \rho Cp\ \delta x/2*\delta t + K/ \delta x$

$C(n)= - K/ \delta x$

$D(n)= \rho Cp\ \delta x/2*\delta t* T_n^{n}$

ITERATIVE SCHEMES

The tridiogonal system for equations above is solved using algorithm. But in the above equations, Q is an unknown parameter, thus the solution of complete problem from x=0 to x=L, cannot be obtained readily because the boundary condition is not known at x=0, but rather an interior temperature history is given. In estimating, one minimizes

$F(Q) = (T_c(x,t)-T_m(x,t))$

Where T_c & T_m are respectively, calculated and measured thermocouple temperatures at (x,t).

The calculated temperature is in general a non-linear function of Q, but it can be solved by using iteration with a linear approximation. Then for the iteration the Taylor series approximation :

$T^{n+1}(Q) = T^{n}(Q) + (Q^{n+1} - Q^{n})\,\partial T/\partial Q$, is used. The subscript is an index related to the number of iteration. The partial derivative in the above equation can be calculated using

$\partial T/\partial Q = T(Q(1+e)-T(Q))/e*Q$

Where e is made equal to 10 % of Q, $\partial t/\partial Q$ is approximated accurately. The temperature on the right hand side of the equation is calculated. The above tridiagonal equations twice with Q & Q(1+e). Using $(\partial F/\partial Q)$, a correction in Q is given by

$Q = -F(Q)/\,\partial T/\partial Q.$

The iteration procedure begins with the estimated value of Q and continues until F is less than say 10 -3.

RESULTS

The slab condition and material properties taken are

L=0.018

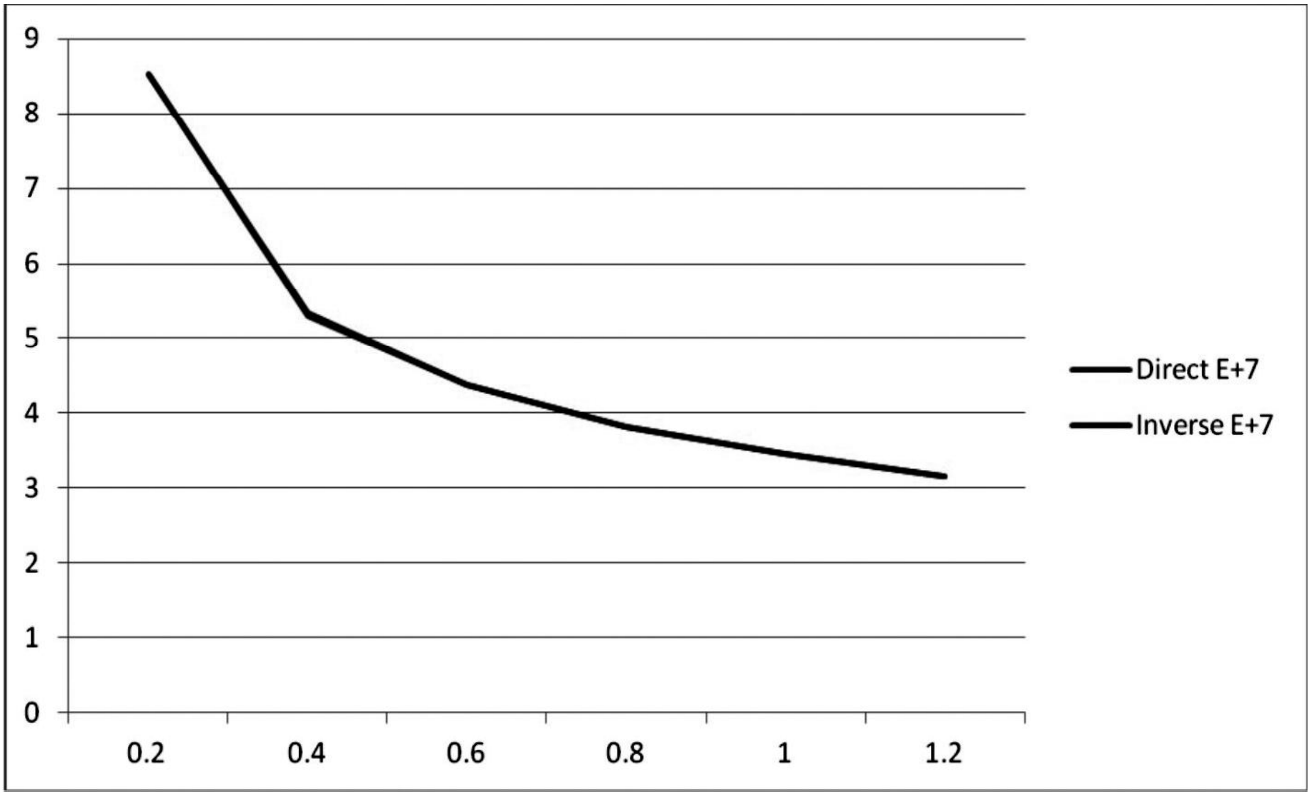
Cp=1750 W sec/Kg. K

Row= 1900 Kg/m, T=300.

Table 1

Time in seconds	Q w/m2 (Direct)	Q w/m2(Inverse)
0.2	8.52 E +007	8.52 E +007
0.4	5.3 E +007	5.33 E +007
0.6	4.37 E +007	4.37 E +007
0.8	3.82 E +007	3.82 E +007
1.0	3.45 E +007	3.45 E +007
1.2	3.16 E +007	3.16 E +007
1.4	2.94 E +007	2.94E +007
1.6	2.76E +007	2.76 E +007
1.8	2.6 E +007	2.6 E +007
2.0	2.46 E +007	2.46 E +007

Figure 1



CONCLUSION & FUTURE WORK

A finite control volume method has been evolved to calculate temperature distribution & heat transfer in a slab of infinite thickness and finite length. An IHCP method was evolved to calculate heat transfer in the slab which is considered as part of rocket nozzle throat. Single temperature sensor, Sensitivity coefficient method has been used to estimate heat transfer in rocket nozzle throat. By assuming some surface heat transfer, the temperature at certain depth below the actual surface had been calculated. This temperature is compared with the measured temperature from a thermo couple, placed at certain depth, and the incremental heat transfer has been calculated. This incremental heat transfer was added to the surface heat transfer. The process was repeated until the measured temperature and calculated temperature at the same depth were equal. In estimating the IHCP problem, past and present time steps had been used.

In the present work, a slab of infinite length and finite thickness is used. The work can be extended to Rocket nozzle throat which is a hollow cylindrical piece with finite thickness, whose outer surface is insulated, with a net heat flux at inner radius. Study of errors in estimation of heat transfer process can also be done.

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