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Fluid Dynamics

PRESSURE DROP AND HEAT TRANSFER TO POWER LAW FLUIDS ACROSS STAGGERED TUBE BANKS

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ABSTRACT

The heat transfer rates and pressure drop data available in the literature for the flow of water and power law non-Newtonian fluid flowing across staggered triangular tube bank are analyzed using capillary tube bundle approach. Correlations for predicting friction factor and heat transfer are proposed and compared with those based on parallel channel model.

Keywords: Heat Transfer, Nusselt number, pseudoshear rate, Reynolds number, tortusoity factor..

INTRODUCTION

Non-Newtonian fluids are encountered in number of process industries; and shell and tube heat exchangers are often used for heating and cooling of these rheologically complex fluids. In the absence of the required ideal tube bank data for these fluids, it is extremely difficult to use modem heat exchanger design techniques. The designs, therefore, are usually based on some reasonable guess made for non-Newtonian fluids derived from Newtonian correlations and curves.

Most of the earlier attempts to correlate the friction factor or heat transfer coefficients and Reynolds number for Newtonian fluids are based on conventional model employing an equivalent diameter and the maximum velocity of the minimum fluid area. *Grimison*[1] *Huge*[2] and *Pierson*[3] correlated their experimental heat transfer data using outside diameter and the maximum velocity in dimensionless groups. *Bergelin* et al.[4, 5 & 6] used volumetric mean diameter to correlate their data in laminar and turbulent range and proposed separate correlations for triangular, staggered square and in line tube arrangements. *Zukauskas*[7] presented an extensive survey of the then available information on Newtonian fluids and recommended correlations for different tube arrangements. *Hughmark*[8] and *Whitaker*[9 &11] successfully predicted heat transfer and pressure drop during flow across pipes or beds of particles in packed or expanded state by assuming the bed to be consisting of a large number of capillary tubes having diameters equals to hydraulic diameter of the bed.

Cruzan[10] and *Adams* and *Bell*[12] reported pressure drop and heat transfer data using aqueous CMC solutions. They used outside tube diameter, maximum velocity and Reed *Metzner Reynolds number*[13] to correlate their data and proposed separate correlations for tube banks of different geometries. Prakash et al.[17] investigated pressure drop and heat transfer with water and aqueous CMC solutions flowing across triangular tube bank. A parallel-plate channel model was used to define the flow in the tube bank and a single valued correlation was proposed for both Newtonian and power law non-Newtonian fluids. *V.K. Mandhani* et al.[18] numerically investigated the heat transfer characteristics for incompressible, steady and Newtonian fluid flow for a bundle of circular cylinders and compared their prediction with that of experimental results of *Bergelin*[6] and that of Le-Clair and Hamielec. Numerical investigation was carried out by *Narasimha Mangadoddy* et al.[19]

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for forced convective heat transfer charactenisties for incompressible power-law fluid past a bundle of circular cylinders and found reasonable agreement of their prediction with the experimental results of *Adams* and *Bell*[12]. *Khan* et al.[20] analytically investigated heat transfer from tube bank in cross-flow under isothermal boundary condition and compared heat transfer results for inline and staggered tube bank. They found higher heat transfer rate for staggered arrangement than that for the inline arrangement. This paper analyses and presents correlations in the light of the capillary tube bundle model.

1. OBJECTIVE

The experimental data have been used to predict the performance of cross-flow tube bank using capillary flow model and also converging-diverging parallel plate channel model. In the capillary flow model approach, the flow across tube bank is considered to be equivalent to that through a collection of tangled capillaries, where capillaries are assumed to be uniform but of non-circular cross-section.

The same experimental data are used to predict the performance of same tube bank using converging-diverging parallel plate channel model to account for the effect of fluid contraction between two adjacent tubes and expansion elsewhere. In this model the flow pattern is assumed to follow a sinusoidal path in the direction of flow and Reynolds number calculation is based on the equivalent diameter and average velocity.

The results obtained from both the above models are compared to obtain better design criterion with a view to explore the possibility of better, unique and more realistic approach.

2. RESEARCH METHODOLOGY

The capillary flow model has already been used by Hughmark (8) and Whitaker (9) for correlating the Newtonian fluid flow and heat transfer data for staggered tube banks. This can be extended for non-Newtonian fluids as well. The flow situation may be considered analogous to a bundle of large number of tortuous capillary tubes of varying cross section with a mean hydraulic radius rH. Defining hydraulic radius as ratio of flow area to wetted perimeter one gets,

$$r_H = \frac{D_o \varepsilon}{4(1-\varepsilon)} \qquad (1)$$

where ε is the void fraction.

The equivalent length Le of tortuous path of the fluid stream, which includes the length of tube bundle and equivalent length due to contraction and expansion and secondary flows, is a function of the actual tube bundle length. It can be represented by $L_e = \beta L$ where β may be termed as tortuous path factor defined as the ratio of equivalent length of tortuous flow stream to actual length of the bank. The parameter β would be equal to π/2 for staggered arrangements and unity for inline arrangements where tube contraction and expansion losses are negligible.

Using those basic concepts, analysis of flow of a power law fluid through capillary tube bundle of hydraulic diameter D_e gives the relationship between wall shear and pseudoshear rate as follows:

$$\tau_w = k\left(\frac{8U}{De}\right)^n = r_H\left(\frac{\Delta P}{Le}\right) = r_H\left(\frac{\Delta P}{\beta L}\right) \qquad (2)$$

$$K' = k\left(\frac{3n+1}{4n}\right) \qquad (3)$$

Defining friction factor as $f_m = \frac{2\tau_w}{\rho U^2}$ (4)

Substituting the value of τ_w, r_H and $\frac{U_s}{\varepsilon}$ in the above equation we get,

$$f_m = \frac{\Delta p\, g_c}{2\rho U_s^2}\,\frac{D_o}{\beta L}\,\frac{\varepsilon^3}{(1-\varepsilon)} \qquad (5)$$

The Reynolds number may now be defined as

$$Re_m = \frac{D_e U_s \rho}{\mu} \qquad (6)$$

$$= \frac{D_o U_s}{(1-\varepsilon)\left(\frac{\tau_w}{k'}\right)^{\frac{n-1}{n}}} \qquad (7)$$

where, $\mu_{eff} = k'(\tau_w . k')^{\frac{n-1}{n}}$ (8)

3. DATA ANALYSIS

The experimental data were from the Ph.D thesis of *Om Prakash*[17]. Om Prakash carried out experimental work on tube bank having 110 copper tubes of 0.95 cm outside diameter arranged on a triangular pitch, 1.43 cm, in eleven longitudinal rows. The test fluids which flowed past the tubes were water and aqueous CMC solutions. Flow characteristic of fluids were determined with the help of capillary viscometer and flow behavior index, n, and consistency index, K, of the fluids were evaluated from pseudoshear plots. Flow behavior indices were reported by authors are 0.73, 0.64, 0.56, 0.77 and 0.61 for 1%, 1.5%, 2%, 3% and 4% CMC solutions respectively.

The overall heat transfer coefficient (Uo) has been calculated from heat balance. The outside heat transfer coefficient for the test fluid (ho) flowing outside the tube bank was calculated using overall and individual resistance evaluating the inside heat transfer coefficient (hi) using Nusselt relation[14] for turbulent flow heat transfer in the entrance region of a tube.

$$\frac{h_i D_i}{K_i} = 0.36\, R_e^{0.8}\, P_r^{0.33} \left(\frac{D_i}{L_e}\right) \text{ for } 10 < L/D_i < 400 \qquad (9)$$

and
$$N_u\, \Delta^{-1/3} = f\left[R_e^{C_1}\, P_r^{C_2} \left(\frac{\mu_{eff}}{\mu_w}\right)^{C_3}\right] \qquad (10)$$

where
$$Nu = \frac{h_o D_o \epsilon}{K(1-\epsilon)} \qquad (11)$$

$$P_r = \frac{C_p\, \mu_{eff}}{K} \qquad (12)$$

$$\Delta = \frac{3n+1}{4n} \qquad (13)$$

For the used data, ratios of effective viscosity at the tube wall and bulk temperature conditions were found to be near unity. *Prandtl numbers* were calculated at bulk fluid temperatures. The exponent on *Prandtl number* is taken as $\frac{1}{3}$ in accordance with boundary layer theory.

The data of *Om Prakash et al.* with those of *Adams* **(15)** and *Bergelin et al.* **(4,5 & 6)** are shown in **Figure.1** where, Nu $Pr^{-1/3}\, \Delta^{-1/3} \left(\frac{1-\varepsilon}{\varepsilon}\right)$ values are plotted against Re_m. For the data included in **Figure.1**, the Reynolds number varies from 0.2 to 0.4, flow behavior index from 0.5 to 1.0, and $\frac{P}{Do}$ from 1.25 to 1.50. All the data points fall almost along a single curve. The regression analysis of the data gives,

$$Nu\, Pr^{-1/3}\, \Delta^{-1/3} \left(\frac{1-\varepsilon}{\varepsilon}\right) = 1.5 + 0.25\, Re_m^{2/3}$$

The above equation correlates the data of *Om Prakash* et al, [17], data of *Adams* [15] and that of *Bergelin* et al *4,5&6* with mean deviation of ± 12.5%.

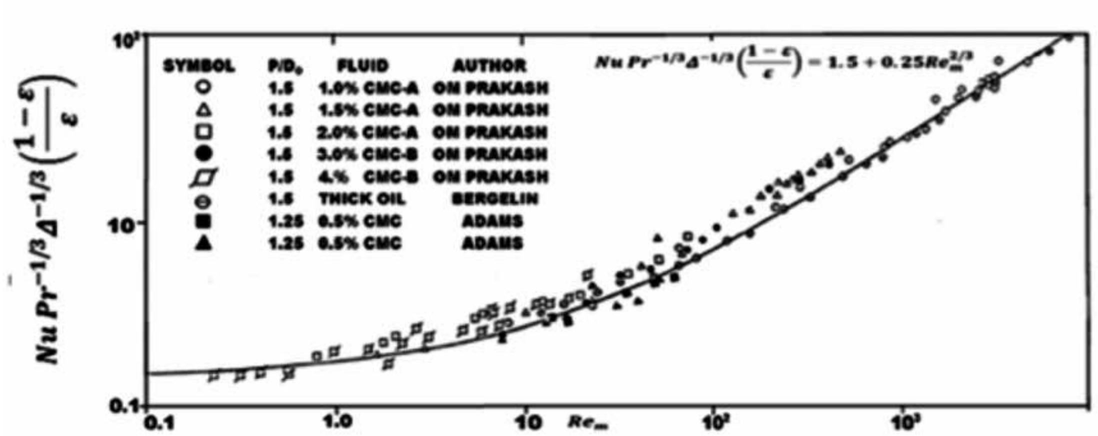

FIG.1 PLOT OF $Nu\, Pr^{-1/3} \Delta^{-1/3} \left(\frac{1-\varepsilon}{\varepsilon}\right)$ Vs REYNOLDS NUMBER FOR TRIANGULAR TUBE ARRANGEMENT (CAPILLARY TUBE MODEL)

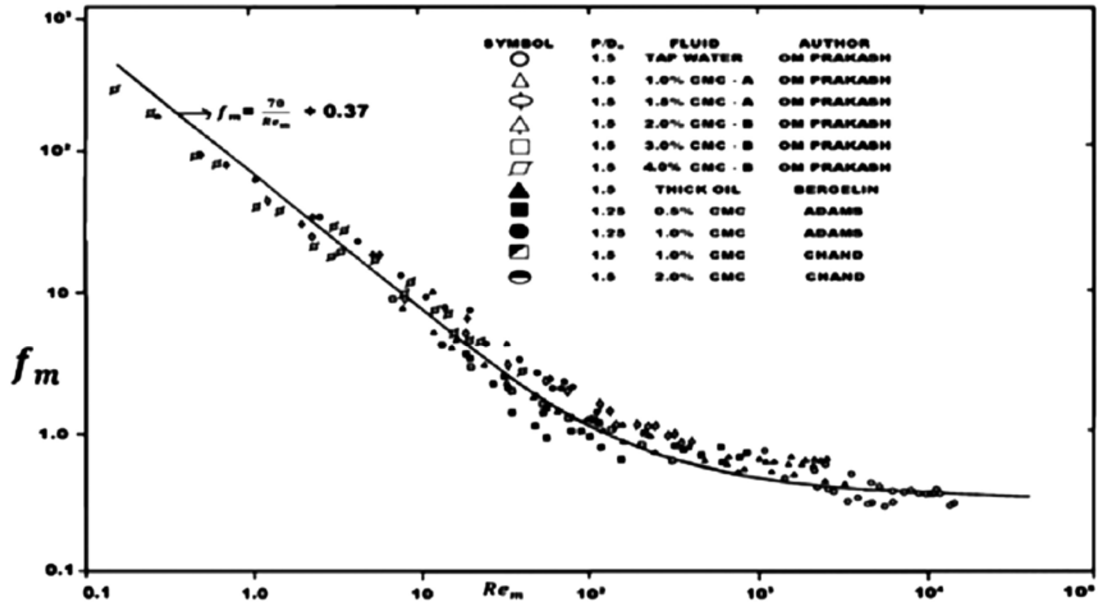

FIG.2 PLOT OF FRICTION FACTOR Vs REYNOLDS NUMBER FOR TRIANGULAR TUBE ARRANGEMENT (CAPILLARY TUBE MODEL)

Figture.2 shows a plot of friction factor versus Reynolds number for triangular tube arrangements having P/D_o = 1.25 and 1.5. At low Reynolds number, f_m is seen to vary as $(Re_m)^{-1}$.

The slope of the curve gradually decreases with increasing Reynolds numbers. Considering the contribution of both, the viscous and form resistance to pressure drop, the data can be correlated by the following equation,

fm = (70/Re¬m)+0.37

which correlates the data of Om Prakash et al. [17] and those of Adams [15], Bergelin et al. [4,5&6] and Chand [16] with mean deviation of ± 25%

Comparison with parallel plate channel model

The correlations for heat transfer and friction factor based on parallel plate channel for triangular tube arrangements were reported by *Prakash* et al. (**Fig.3 to 5**). These are compared with equations obtained based on capillary tube bundle model.

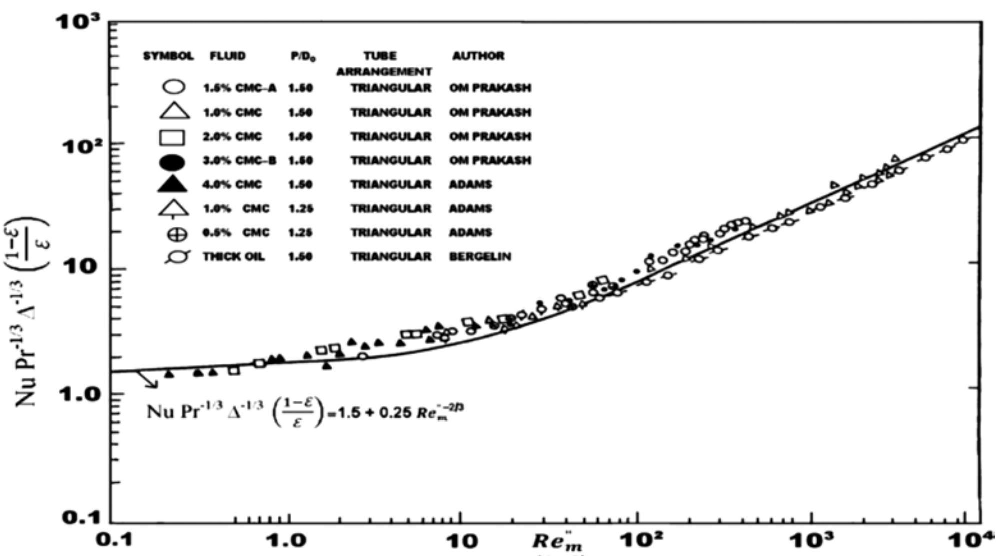

Fig.3 PLOT OF $Nu\,Pr^{-1/3}\,\Delta^{-1/3}\left(\frac{1-\varepsilon}{\varepsilon}\right)$ Vs REYNOLDS NUMBER

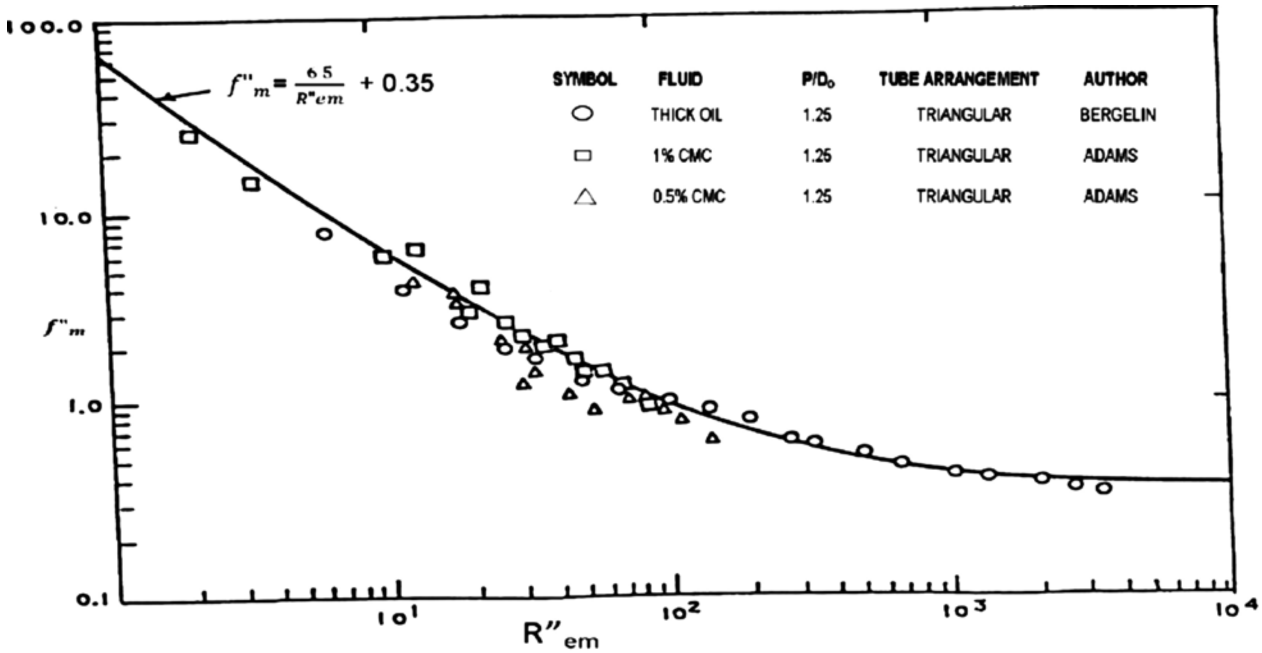

FIG.4 PLOT OF FRICTION FACTOR Vs REYNOLDS NUMBER

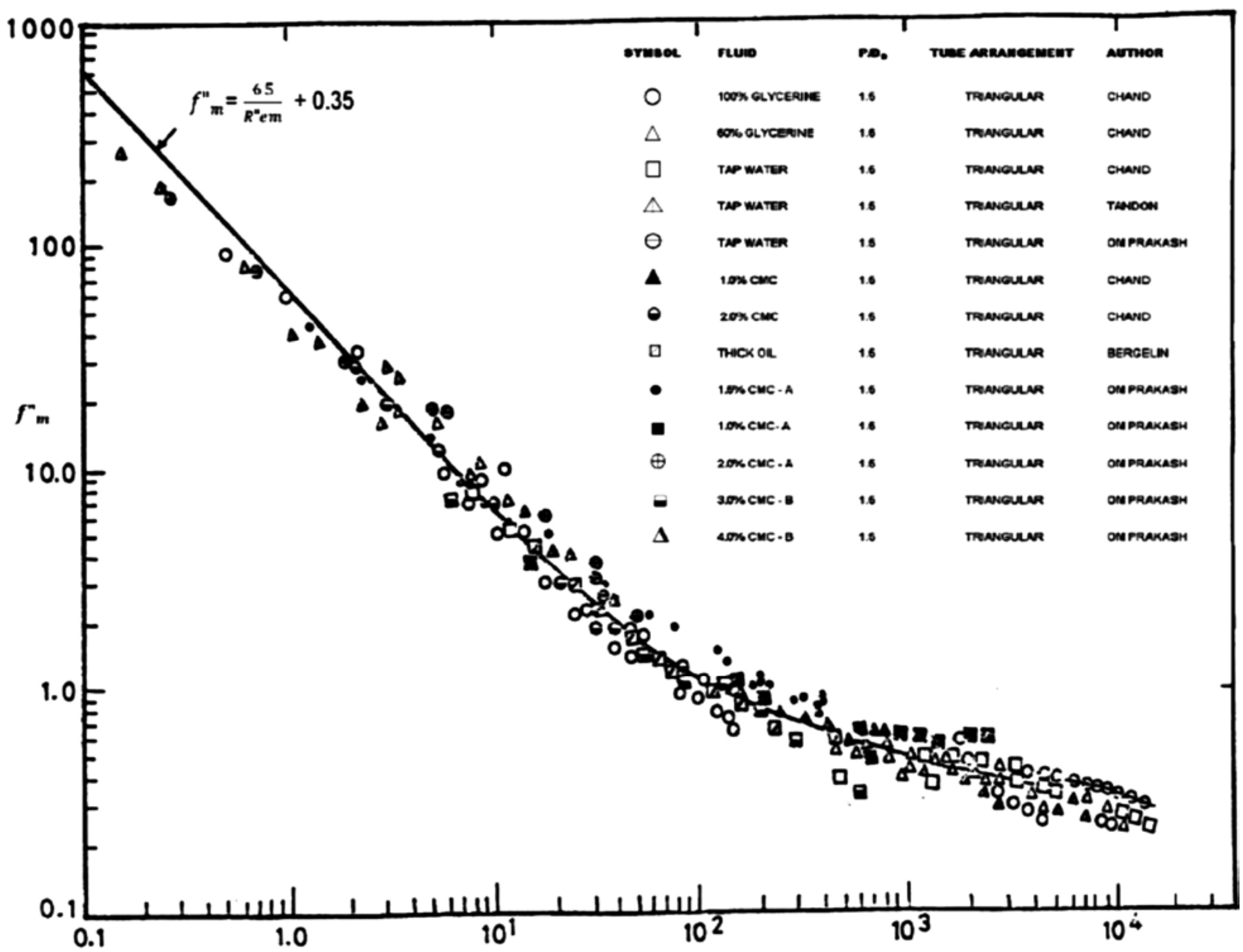

FIG.5 PLOT OF FRICTION FACTOR VERSUS REYNOLDS NUMBER

The equations based on parallel plate channel model gives lower deviations (±6%) for heat transfer and ±14.0% for pressure drop respectively than those based upon capillary tube model. 3. Conclusions

$f_m = \frac{16\beta\beta_f}{Rem}$ for capillary tube bundle model and

$f_m = \frac{24\beta\beta_f}{Rem}$ for parallel plate channel model

where $\beta = \pi/2$ for triangular tube arrangements and

β_f = form factor

The values of form factor (β_f) were found as 2.787 and 1.725 for capillary tube bundle and parallel plate channel model respectively. Thus the form factor in the case of capillary tube bundle model is 1.616 times more than the parallel plate channel model. The parameter β_f is form factor which accounts for form resistance and is the main cause of increase in drag.

A bank of tube can be represented either by a set of capillary tube bundle or by a set of parallel plate channels having hydraulic diameter equal to volumetric hydraulic diameter of the tube bank. Use of dimensionless numbers Nu, Pr and Re defined on the basis of both the approaches gives single valued correlation for both Newtonian and power law non-Newtonian fluids. However, the correlation based on parallel plate channel approach is found to be better than that of the capillary tube bundle approach.

4. Acknowledgement

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D_e	hydraulic diameter of tube bank $\frac{D_o\epsilon}{1-\epsilon}$, m.
D_i	inside tube diameter, m
D_o	outside tube diameter, m
f_m	friction factor based on capillary tube model $(D\,\Delta p\, g_c\, \epsilon^3)/2\rho\, U_s^2\,(1-\epsilon)$
f'_m	friction factor based on parallel plate channel model $(D\,\Delta p\, g_c\, \epsilon^3)/2\rho\, U_s^2\,(1-\epsilon)$
h_i	heat transfer coefficient of the fluid flowing inside the tube, w/m² °K
h_o	heat transfer coefficient of fluid flowing outside the tube w/m² °K
K	thermal conductivity w / m K
K'	power law consistency constant based on capillary tube bundle model K, (3n+1) / 4n, Kg/m Sec^{2-n}
K"	power law consistency constant based on parallel plate channel model K (2n + 1) / 3n, Kg/m Sec^{2-n}
L	length of the tube bank, m
n	flow behavior index

$\tilde{Nu}$	Nusselt number
Δp	differential pressure drop, N/m^2
Re_m	Reynolds number based on capillary bundle tube model $D_o U_s \rho / K' (\tau_w/K')^{\frac{n-1}{n}} (1-\varepsilon)$
$Re_{m''}$	Reynolds number based on parallel plate channel model $D_o U_s \rho/K'' (\tau_w/K'')^{\frac{n-1}{n}} (1-\epsilon)$
H	hydraulic radius of tube bank $\frac{D_0 \epsilon}{4(1-\epsilon)}$
U	average velocity, m/s
U_s	superficial velocity, m/s
β	tortusoity factor
Δ	$(\frac{3n+1}{4n})$ for capillary tube bundle model
Δ	$\frac{2n+1}{3n}$ for parallel plate channel model
ϵ	void fraction
ρ	density, Kg/m^3
τ_w	shear stress at wall, N/m^2
μ_w	viscosity at wall temperature, Ns/m^2
μ_{eff}	effective viscosity based on capillary tube model, $K' (\tau_w/K')^{\frac{n-1}{n}}$, Ns/m^2
μ_{eff}	effective viscosity based on parallel plate tube model, $K'' (\tau_w/K'')^{\frac{n-1}{n}}$, Ns/m^2

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